Which pieces anchor the shape from shading puzzle and how they fit together

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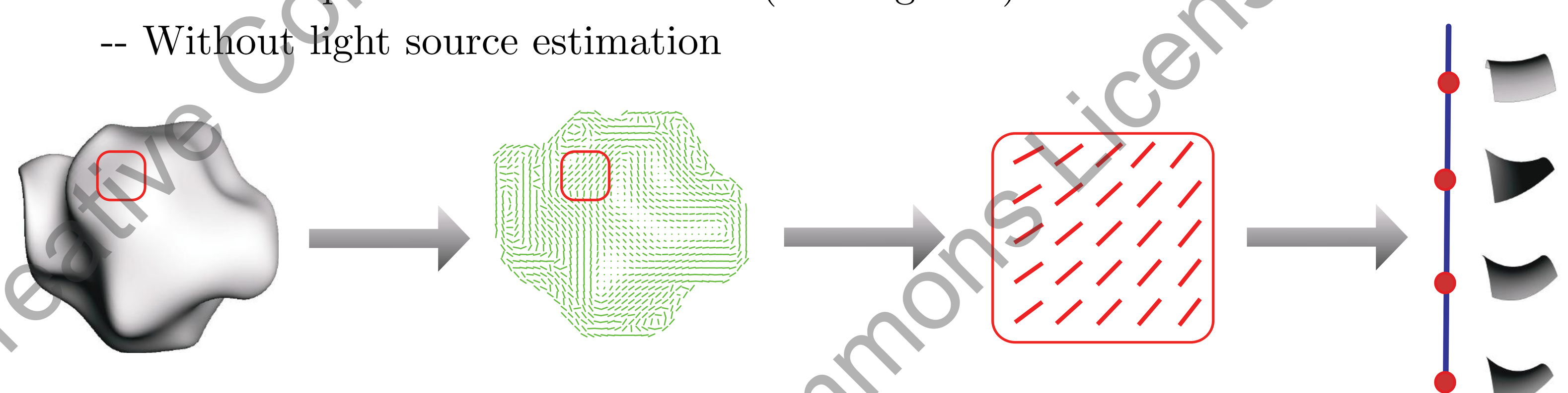
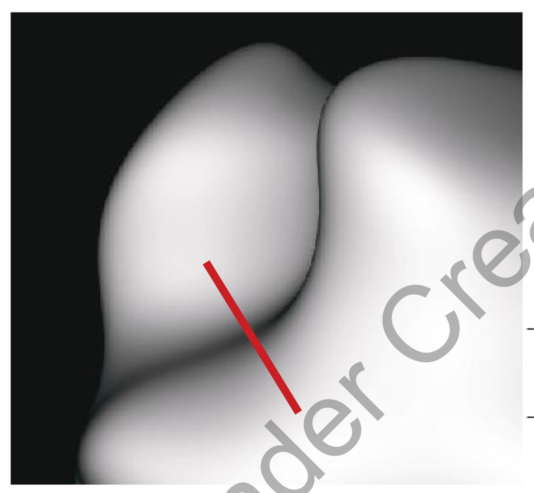
SFS on Ridges

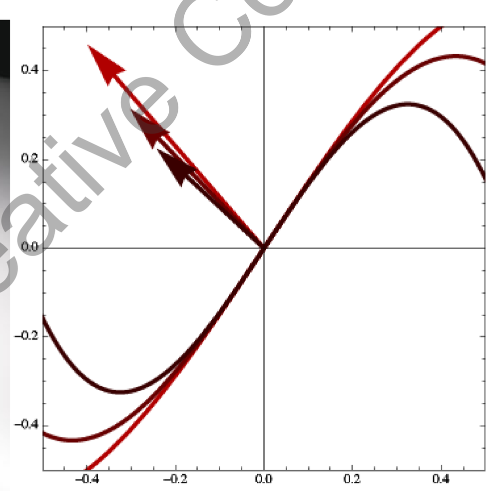
Shape from Shading
-- Based on patterns of orientations (shading flow)
-- Without light source estimation



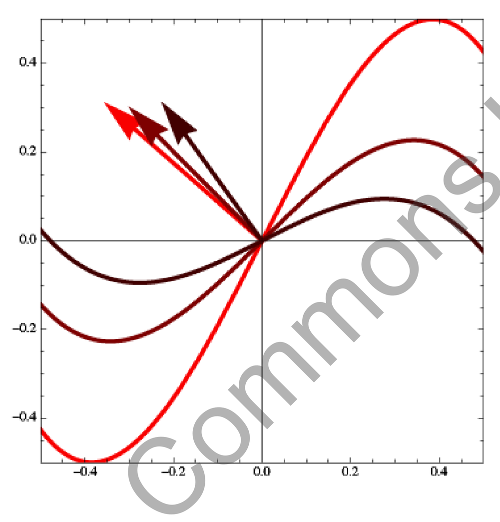
-- Complexity vanishes along anisotropic features (ridges)



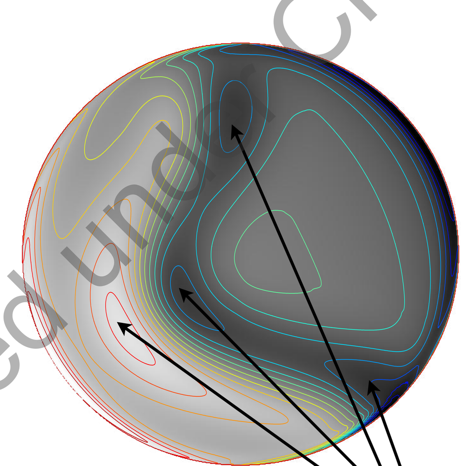
A Lambertian shaded surface with a marked ridge. The tangent plane and light source are unknown.

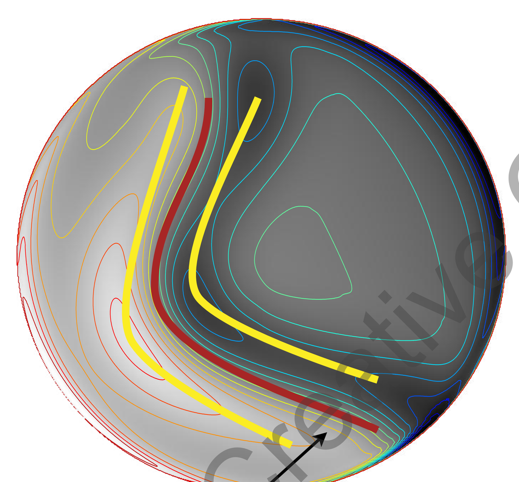


Solutions for the cross section as the assumed light source changes.



Solutions for the cross section as the tangent plane changes



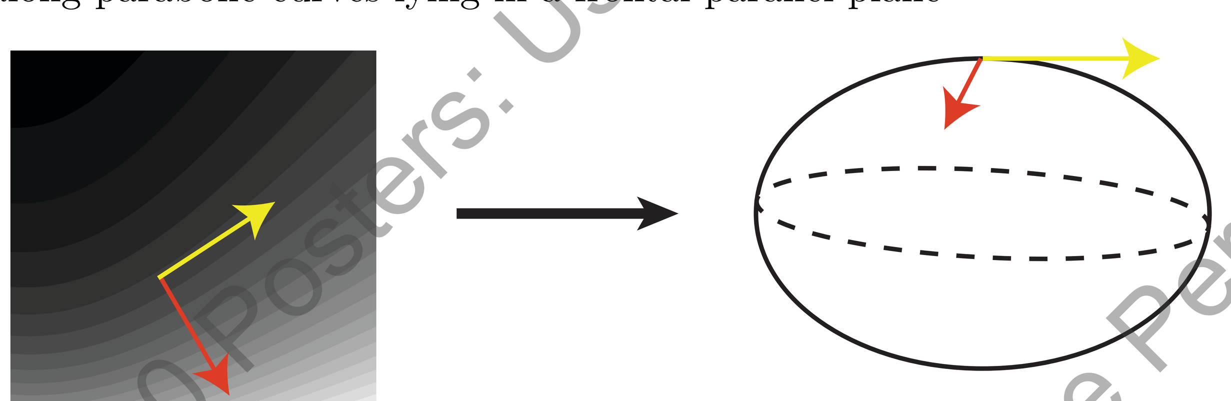


Critical Points $\nabla I = 0$
become Critical Contours $\nabla I < \epsilon$

For frontal parallel ridges with hemispheric lighting, the image gradient and isophote directions are the eigenvectors of the view dependent shape operator: $d\tilde{N} : I \to S^2$ (roughly matrix of surface curvatures). We compare the eigendecomposition of the outer product of the brightness gradient to the Singular Value Decomposition of $d\tilde{N}$

$$E[\nabla I \otimes \nabla I] = V^T \Sigma V = V^T \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} V, \qquad d\tilde{N} = U \Sigma V^T$$

For these to be equal, we need $U\Sigma = V\Sigma$ which happens at frontal parallel points or along parabolic curves lying in a frontal parallel plane

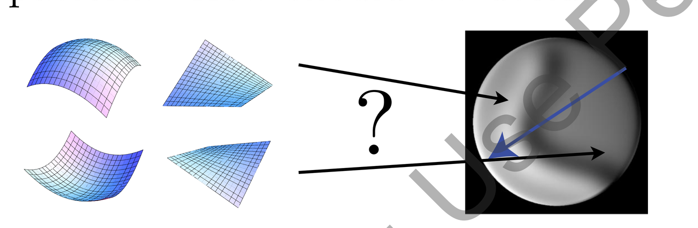


The surface can thus be described, e.g. by an ellipsoidal surface with the axis along the isophote and image gradient at these Critical Contours.

Concave/Convex Ambiguity

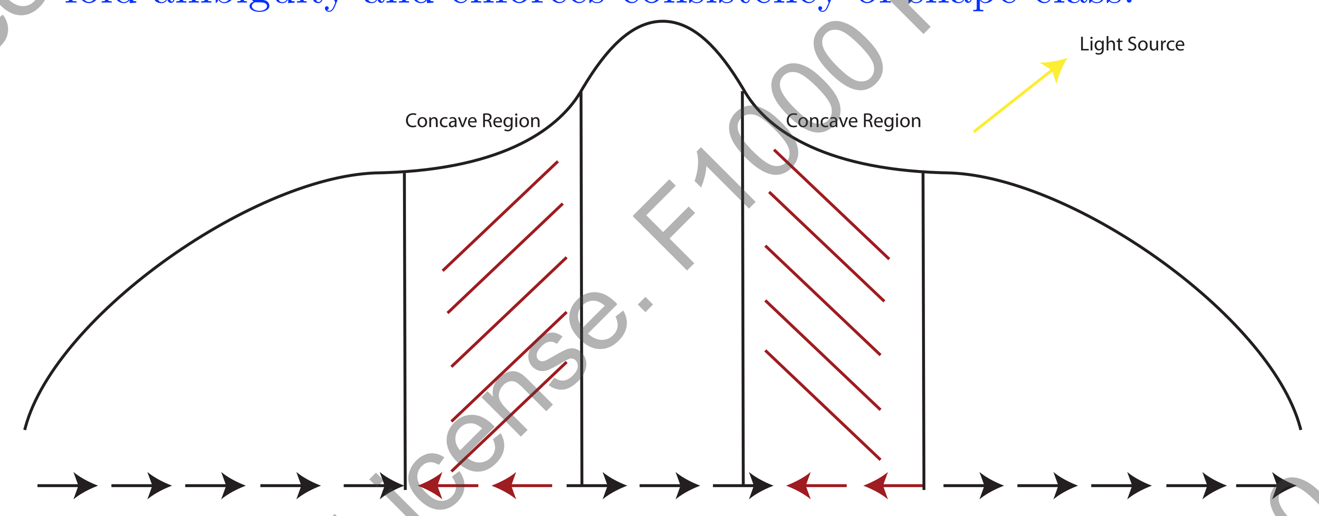
The concave/convex ambiguity cannot be solved through purely local computation. As shown in our previous poster (VSS 2012), there are always at least four possible local surfaces no matter the shading pattern. How should we decide which of these four is the correct one?

$$||\nabla I|| = \lambda_1 \langle l_t, \vec{u} \rangle$$



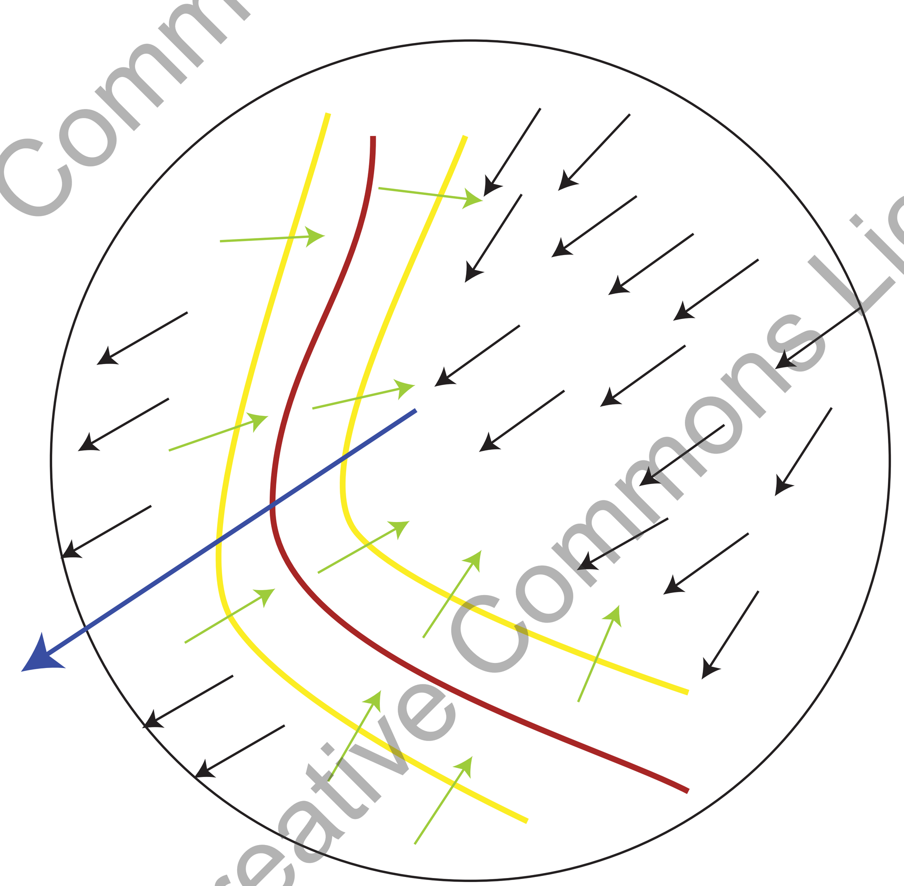
Since $||\nabla I|| > 0, \lambda_1$ and $\langle l_t, \vec{u} \rangle$ must share the same sign.

At a global scale, there often emerges "an effective light source", l_t. This leads to a large scale gradient. This large scale gradient resolves local 4-fold ambiguity and enforces consistency of shape class.

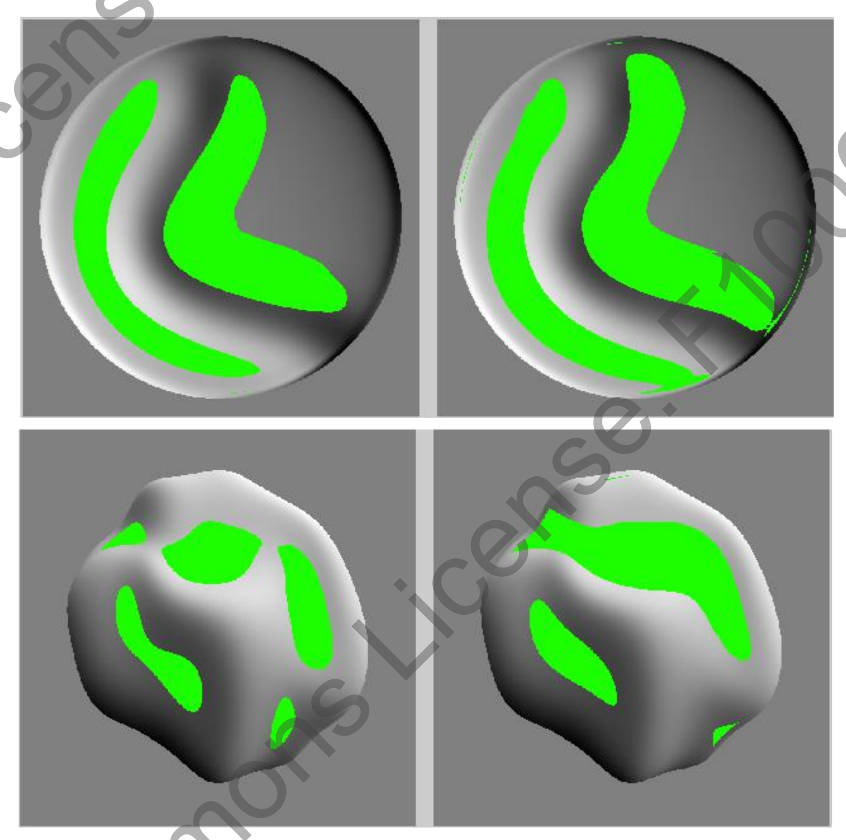


1D Version: If the local brightness gradient (arrows) point in opposite direction (red regions) of the global gradient, then the region is concave.

Signs of Largest Eigenvalue



2D Version: If the angle between the large scale gradient (large blue vector) and the local gradients (each individual green vector) is greater than 90 degs, then the region is concave.



True signs of maximum eigenvalue, λ_1

Recovered signs of maximum eigenvalue, λ_1

We solve the depths from our curvature tensor by minimizing the following functional and solve using a series of Poisson P.D.Es.

$$\text{Minimize} \int \left(d\tilde{N} - E[\nabla I \otimes \nabla I] \right) dA \text{ for } f(x,y)$$

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Results (Ridge Features)

We use simple surfaces with a prominent ridge -- the majority of the image structure is a ridge. Our lighting involves two separate sources, but we do not use the assumption anywhere.